Course: Time Series Analysis

Name:……………………………………………..

Group:……………………………………………

Final Exam

Year	Q	Y	MA4.	Centered MA	SI	Detrend and Deseanlize
2020	Q1	100				
	Q2	120	126.5			
	Q3	160	129.5	128	129.43	0.97
	Q4	126	137	133.25	92.25	1.03
2021	Q1	112	143.5	140.25	77.45	1.03
	Q2	150	144	143.75	100.86	1.03
	Q3	186	145	144.5	129.43	0.99
	Q4	128	145.5	145.25	92.25	0.96
2022	Q1	116	147	146.25	77.45	1.02
	Q2	152	148	147.5	100.86	1.02
	Q3	192	149.5	148.75	129.43	1.00
	Q4	132	152.5	151	92.25	0.95
2023	Q1	122	158	155.25	77.45	1.01
	Q2	164	162.5	160.25	100.86	1.01
	Q3	214				
	Q4	150				

Exercise 1

The following table presents the quarterly sales of washing machines for the period between 2020-2023:

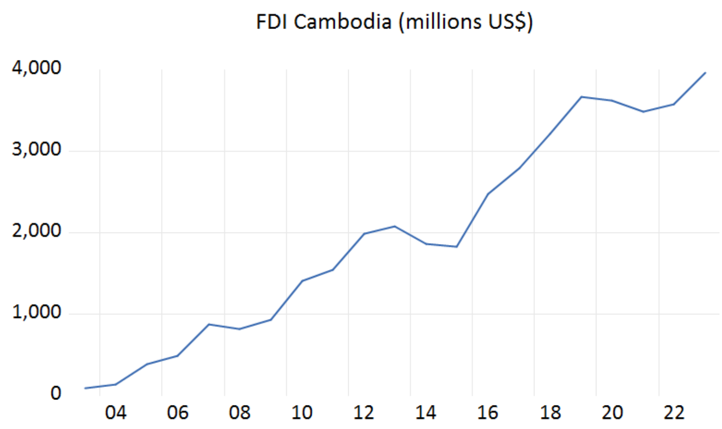
1. Calculate the trend values using **MA4**.(1.5pt)
2. Compute the seasonal indices for the data using the **simple average method**.(1pt)
3. **Detrend** and **deseasonalize** the time series using the **multiplicative** model.(1.5pt)
4. Estimate the quarterly sales of 2024, assuming that the estimated annual sales for 2024 is 680 (1pt)

	Q1	Q2	Q3	Q4	
2020	100	120	160	126	506
2021	112	150	186	128	576
2022	116	152	192	132	592
2023	122	164	214	150	650
$\bar{Q}$	112.5	146.5	188	134	145.25
SI	77.45	100.86	129.43	92.25	
	170	170	170	170	680
$\hat{Q}_{2024}$	131.67	171.46	220.03	156.83	

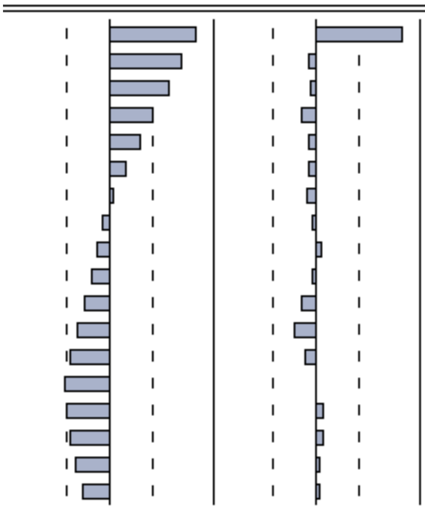
Exercise 2 (13pt): The following table represents foreign direct investment net inflows (in millions US $) in Cambodia:

Years	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
FDI	81.6	131.4	379.2	483.2	867.3	815.2	928.4	1404.3	1538.9	1988.1	2068.5
Years	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**-**
FDI	1853.5	1822.8	2475.9	2788.1	3212.6	3663.0	3624.6	3483.5	3578.8	3958.8	-

The time series plot , along with the corresponding Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) are represented below:



Sample: 2003 2023
Included observations: 21



	AC	PAC	Q-Stat
1	0.857	0.857	17.757
2	0.717	-0.068	30.838
3	0.586	-0.050	40.037
4	0.437	-0.147	45.459
5	0.297	-0.070	48.120
6	0.165	-0.079	48.997
7	0.038	-0.090	49.046
8	-0.068	-0.038	49.216
9	-0.132	0.045	49.917
10	-0.182	-0.029	51.366
11	-0.247	-0.150	54.312
12	-0.333	-0.211	60.273
13	-0.406	-0.104	70.245
14	-0.445	0.003	83.890
15	-0.439	0.082	99.436
16	-0.403	0.067	115.16
17	-0.349	0.040	129.83
18	-0.274	0.032	141.92

1- From visual inspection, is the series stationary? (0.5pt). **Non-stationary time series**
2- Why? (0.5pt) **It has a non-constant mean**
3- What kind of trend exhibit in this series? (0.5pt) **Deterministic trend**
4- Based on the trend type, what is the most appropriate technique for making the time series stationary (0.5pt) **Detrending by model fitting**
5- The previous data set can be summarized by the following: n=21, $\sum x = 231$, $\sum y = 41147.7, \bar{x} = 11, \bar{y} = 1959.4$, $\sum xy = 609095.5$, $\sum x^2 = 3311$. Transform segment of the preceding series into a stationary series (by trend fitting model), then compute the residuals.

Years	FDI (y)	x (coded time)	xy	x^2	$\hat{y}$ (1.5pt)	Residuals ϵ_t (1.5pt)
2003	81.6	1	81.6	1	/	/
2004	131.4	2	262.8	4	/	/
2005	379.2	3	1137.6	9	333.72	45.48
2006	483.2	4	1932.8	16	/	/
2007	867.3	5	4336.5	25	740.14	127.16
2008	815.2	6	4891.2	36	/	/
2009	928.4	7	6498.8	49	1146.56	-218.16
2010	1404.3	8	11234.4	64	1349.77	54.53
2011	1538.9	9	13850.1	81	/	/
2012	1988.1	10	19881.0	100	/	/
2013	2068.5	11	22753.5	121	1959.4	109.1
2014	1853.5	12	22242.0	144	2162.61	-309.11
2015	1822.8	13	23696.4	169	/	/
2016	2475.9	14	34662.6	196	2569.03	-93.13
2017	2788.1	15	41821.5	225	2772.24	15.86
2018	3212.6	16	51401.6	256	2975.45	237.15
2019	3663.0	17	62271.0	289	3178.66	484.34
2020	3624.6	18	65242.8	324	/	/
2021	3483.5	19	66186.5	361	/	/
2022	3578.8	20	71576.0	400	3788.29	-209.49
2023	3958.8	21	83134.8	441	3991.5	-32.7
	$\bar{y}$	$\bar{x}$	$\sum xy$	$\sum x^2$		
	1959.4	11	609095.5	3311		

b (0.5pt)	$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{609095.5 - 21(11)(1959.4)}{3311 - 21(11)^2} = \frac{156474.1}{770} = 203.21$
a (0.5pt)	$a = \bar{y} - b\bar{x} = 1959.4 - (203.21)(11) = -275.91$
EQUATION (0.5pt)	$\hat{y} = a + bx = -275.91 + 203.21x$

6- Estimate 2nd ,7th, and 9th order sample autocorrelation coefficients , then determine their significance. (1.5pt)

Coefficents	Significance
$r_2 = 0.717$	Significant
$r_7 = 0.038$	Not-significant
$r_9 = -0.132$	Not-significant

7- Estimate 3rd, 8th and 9th order sample partial autocorrelation coefficients, then determine their significance.(1.5pt)

Coefficents	Significance
$r_{33} = -0.050$	Not-significant
$r_{88} = -0.038$	Not-significant
$r_{99} = 0.045$	Not-significant

After transforming the time series, a test was performed on the residuals, and the outcomes are illustrated in the table below:

Null Hypothesis: RESIDUALS has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on SIC, maxlag=4)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.623337	0.0011
Test critical values:	1% level	-2.692358	
	5% level	-1.960171	
	10% level	-1.607051	

8- What is the name of the test, and what is it used for? (1pt)**Augmented Dickey Fuller Test (ADF), It is used to assess the stationarity of the time series statistically.**

9- Interpret the outcomes of the test according to the critical value approach? (2pt)

Null hypothesis (H0): The residuals time series is not stationary due to the presence of a unit root.

Alternative hypothesis (Ha): The residuals time series is stationary due to the absence of a unit root.

Based on the critical value approach $ADFstat. = |-3{,}623| > critical - value_{\alpha=0.05} = |-1.960|$, thus we reject the null hypothesis at a 5% level of significance, and the residuals time series is **stationary** due to the absence of a unit root.

10- What is the name of the series that becomes stationary by this type of transformation? (0.5pt) **Trend stationary time series**

Exercise 3 (2pt)

Assume that a sales series revealed through the ACF and PACF charts that the fitting model is AR(2), with c=42, $\phi_1 = -0.49$ and $\phi_2 = 0.12$.

1- Determine the model equation.(1pt) $AR(2): Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t = 42 - 0.49Y_{t-1} + 0.12Y_{t-2}$

2- Assume that $Y_{2023} = 30$ and $Y_{2024} = 26$, estimate the sales of 2025 and 2026 (1pt)

Years	Sales	Forecasts
2023	30	-
2024	26	-
2025	-	$\hat{y}_{2025} = 42 - 0.49Y_{2024} + 0.12Y_{2023} = 42 - 0.49(26) + 0.12(30) = 32.86$
2026	-	$\hat{y}_{2026} = 42 - 0.49\hat{Y}_{2025} + 0.12Y_{2024} = 42 - 0.49(32.72) + 0.12(26) = 29.02$

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Final Exam

Year	Q	Y	MA4.	Centered MA	SI	Detrend and Deseanalize
2020	Q1	96				
	Q2	120	125.5			
	Q3	160	129.5	127.5	129.76	0.97
	Q4	126	137	133.25	92.15	1.03
2021	Q1	112	143.5	140.25	76.96	1.04
	Q2	150	144	143.75	101.12	1.03
	Q3	186	145	144.5	129.76	0.99
	Q4	128	145.5	145.25	92.15	0.96
2022	Q1	116	147	146.25	76.96	1.03
	Q2	152	148	147.5	101.12	1.02
	Q3	192	149.5	148.75	129.76	0.99
	Q4	132	152.5	151	92.15	0.95
2023	Q1	122	158	155.25	76.96	1.02
	Q2	164	162	160	101.12	1.01
	Q3	214				
	Q4	148				

Exercise 1 (5 pt)

The following table presents the quarterly sales of washing machines for the period between 2020-2023:

1. Calculate the trend values using **MA4** (1.5pt)
2. Compute the seasonal indices for the data using the **simple average method**. (1pt)
3. **Detrend** and **deseasonalize** the time series using the **multiplicative** model.(1.5pt)
4. Estimate the quarterly sales of 2024, assuming that the estimated annual sales for 2024 is 700 (1pt)

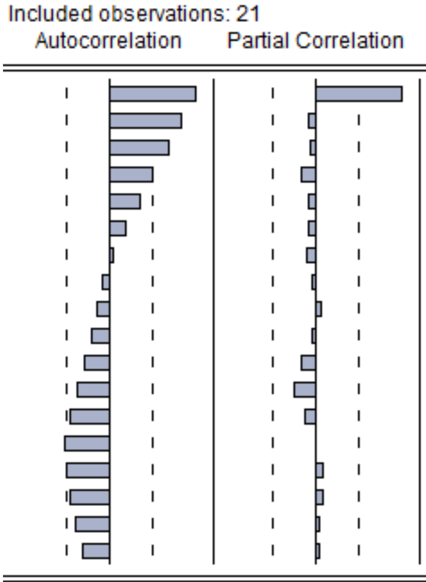
	Q1	Q2	Q3	Q4	
2020	96	120	160	126	502
2021	112	150	186	128	576
2022	116	152	192	132	592
2023	122	164	214	148	648
$\bar{Q}$	111.5	146.5	188	133.5	144.88
SI	76.96	101.12	129.76	92.15	
	175	175	175	175	700
$\hat{Q}_{2024}$	134.68	176.96	227.08	161.26	

Exercise 2 (13pt): The following table represents foreign direct investment net inflows (in millions US $) in Cambodia:

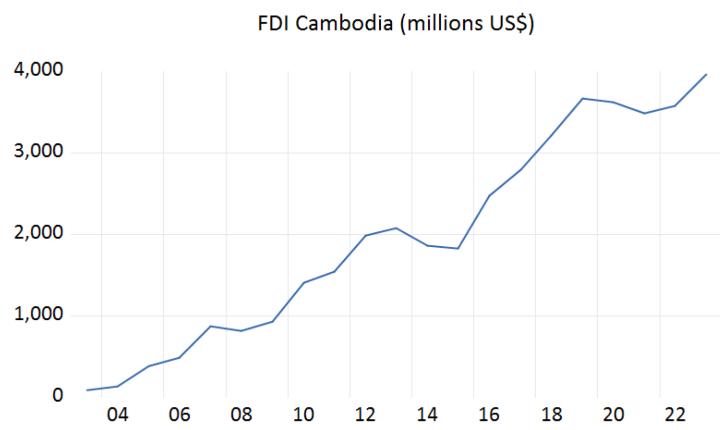
Years	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
FDI	81.6	131.4	379.2	483.2	867.3	815.2	928.4	1404.3	1538.9	1988.1	2068.5
Years	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**-**
FDI	1853.5	1822.8	2475.9	2788.1	3212.6	3663.0	3624.6	3483.5	3578.8	3958.8	-

The time series plot, along with the corresponding Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) are represented below:

Sample: 2003 2023
Included observations: 21



	AC	PAC	Q-Stat
1	0.857	0.857	17.757
2	0.717	-0.068	30.838
3	0.586	-0.050	40.037
4	0.437	-0.147	45.459
5	0.297	-0.070	48.120
6	0.165	-0.079	48.997
7	0.038	-0.090	49.046
8	-0.068	-0.038	49.216
9	-0.132	0.045	49.917
10	-0.182	-0.029	51.366
11	-0.247	-0.150	54.312
12	-0.333	-0.211	60.273
13	-0.406	-0.104	70.245
14	-0.445	0.003	83.890
15	-0.439	0.082	99.436
16	-0.403	0.067	115.16
17	-0.349	0.040	129.83
18	-0.274	0.032	141.92



1- From visual inspection, is the series stationary? (0.5pt). **Non-stationary time series**
2- Why? (0.5pt) **It has a non-constant mean**
3- What kind of trend exhibit in this series? (0.5pt) **Deterministic trend**
4- Based on the trend type, what is the most appropriate technique for making the time series stationary (0.5pt) **Detrending by model fitting**
5- The previous data set can be summarized by the following: n=21, $\sum x = 231$, $\sum y = 41147.7, \bar{x} = 11, \bar{y} = 1959.4$, $\sum xy = 609095.5$, $\sum x^2 = 3311$. Transform segment of the preceding series into a stationary series (by trend fitting model), then compute the residuals.

Years	FDI (y)	x (coded time)	xy	x^2	$\hat{y}$ (1.5pt)	Residuals ϵ_t (1.5pt)
2003	81.6	1	81.6	1	-72.7	154.3
2004	131.4	2	262.8	4	/	/
2005	379.2	3	1137.6	9	333.72	45.48
2006	483.2	4	1932.8	16	536.93	-53.73
2007	867.3	5	4336.5	25	/	/
2008	815.2	6	4891.2	36	/	/
2009	928.4	7	6498.8	49	/	/
2010	1404.3	8	11234.4	64	1349.77	54.53
2011	1538.9	9	13850.1	81	1552.98	-14.08
2012	1988.1	10	19881.0	100	/	/
2013	2068.5	11	22753.5	121	1959.4	109.1
2014	1853.5	12	22242.0	144	2162.61	-309.11
2015	1822.8	13	23696.4	169	/	/
2016	2475.9	14	34662.6	196	2569.03	-93.13
2017	2788.1	15	41821.5	225	2772.24	15.86
2018	3212.6	16	51401.6	256	/	/
2019	3663.0	17	62271.0	289	3178.66	484.34
2020	3624.6	18	65242.8	324	/	/
2021	3483.5	19	66186.5	361	3585.08	-101.58
2022	3578.8	20	71576.0	400	/	/
2023	3958.8	21	83134.8	441	3991.5	-32.7
	$\bar{y}$	$\bar{x}$	$\sum xy$	$\sum x^2$	/	/
	1959.4	11	609095.5	3311	/	/
b (0.5pt)	$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{609095.5 - 21(11)(1959.4)}{3311 - 21(11)^2} = \frac{156474.1}{770} = 203.21$					
a (0.5pt)	$a = \bar{y} - b\bar{x} = 1959.4 - (203.21)(11) = -275.91$					
EQUATION (0.5pt)	$\hat{y} = a + bx = -275.91 + 203.21x$					

6- Estimate 3rd ,8th, and 10th order sample autocorrelation coefficients , then determine their significance. (1.5pt)

Coefficents	Significance
$r_3 = 0.586$	Significant
$r_8 = -0.068$	Not-significant
$r_{10} = -0.182$	Not-significant

7- Estimate 1st, 6th and 7th order sample partial autocorrelation coefficients, then determine their significance.(1.5pt)

Coefficents	Significance
$r_{11} = 0.857$	Significant
$r_{66} = -0.079$	Not-significant
$r_{77} = -0.090$	Not-significant

After transforming the time series, a test was performed on the residuals, and the outcomes are illustrated in the table below:

Null Hypothesis: RESIDUALS has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on SIC, maxlag=4)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.623337	0.0011
Test critical values:	1% level	-2.692358	
	5% level	-1.960171	
	10% level	-1.607051	

8- What is the name of the test, and what is it used for?(1pt) **Augmented Dickey Fuller Test (ADF), It is used to assess the stationarity of the time series statistically.**

9- Interpret the outcomes of the test according to the p-value approach? (2pt)

Null hypothesis (H0): The residuals time series is not stationary due to the presence of a unit root.

Alternative hypothesis (Ha): The residuals time series is stationary due to the absence of a unit root.

Based on the p-value Approach $p-value = 0.0011 < \alpha = 0.05$, thus we reject the null hypothesis at a 5% level of significance, and the residuals time series is **stationary** due to the absence of a unit root.

10- What is the name of the series that becomes stationary by this type of transformation? (0.5pt) **Trend stationary time series.**

Exercise 3 (2pt)

Assume that a sales series revealed through the ACF and PACF charts that the fitting model is AR(2), with c=40, $\phi_1 = -0.48$ and $\phi_2 = 0.13$.

1- Determine the model equation. (1pt) $AR(2): Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t = 40 - 0.48Y_{t-1} + 0.13Y_{t-2}$

2- Assume that $Y_{2023} = 32$ and $Y_{2024} = 28$, estimate the sales of 2025 and 2026. (1pt)

Years	Sales	Forecasts
2023	32	-
2024	28	-
2025	-	$\hat{y}_{2025} = 40 - 0.48Y_{2024} + 0.13Y_{2023} = 40 - 0.48(28) + 0.13(32) = 30.72$
2026	-	$\hat{y}_{2026} = 40 - 0.48\hat{Y}_{2025} + 0.13Y_{2024} = 40 - 0.48(30.72) + 0.13(28) = 28.89$